Equation of Forces: A Unified Action for Electromagnetism, Weak, Strong, and Gravity

Private Foundation for Theoretical Physics

With contributions by **Dr. G. N. R. Tripathi**

August 23, 2025

Abstract

We formulate a single-action unification of the four known fundamental interactions—electromagnetism, weak, strong, and gravity—in a four-dimensional framework that remains mathematically consistent from laboratory scales to the Planck regime. The action combines non-Abelian Yang–Mills dynamics for the Standard Model (SM) gauge group with higher-curvature extensions of General Relativity, enabling both a viable classical limit and a renormalization-group (RG) flow suggestive of ultraviolet (UV) asymptotic safety. We present explicit variational derivations of the field equations, discuss gauge fixing and BRST structure, analyze coupled RG flows including gravitational corrections, and delineate linked experimental predictions such as gauge coupling unification windows, proton decay expectations (under GUT embeddings), vacuum stability, and inflationary connections through the R^2 sector. The *Equation of Forces* (EoF) constitutes the most explicit four-dimensional, single-action candidate for unifying all interactions without logical contradiction. This article focuses on forces; companion work extends the framework to matter, neutrinos, axions, dark sectors, and cosmology.

Keywords: Force Unification, Asymptotic Safety, Higher-Curvature Gravity, Yang–Mills, Renormalization Group, Proton Decay, Vacuum Stability

1. Introduction

The drive to unify nature's forces has punctuated the history of physics: Newton's universal gravitation, Maxwell's unification of electricity and magnetism, the electroweak theory of Glashow–Salam–Weinberg, and QCD for the strong interaction. Yet, reconciling gravity with quantum field theory in the same practical framework remains a central challenge. In this work we construct a single four-dimensional action that houses the *four forces* coherently and can be analyzed within the functional renormalization group (FRG) to probe UV consistency.

We emphasize: (i) a unified variational principle for all force equations; (ii) a concrete higher-curvature gravitational sector compatible with classical tests and early-universe phenomenology; (iii) renormalization with gravity-induced deformations of gauge β-functions; (iv) predictive bridges to observables (gauge unification, proton decay under GUT completions, inflationary scales from R^2). Matter and cosmology extensions are treated in the companion EoE article.

2. Foundations and Notation

We work on a Lorentzian manifold $(\mathcal{M}, g_{\mu\nu})$ with signature $(-,+,+,+)$, Levi-Civita connection, and Riemann tensor $R^{\rho}{}_{\sigma\mu\nu}$. The SM gauge group is $G_{\rm SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$ with gauge fields $\mathcal{A}_\mu = \{G^A_\mu, W^I_\mu, B_\mu\}$ and field strengths

$$G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_s f^{ABC} G^B_\mu G^C_\nu, \tag{1}$$

$$W^I_{\mu\nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g\epsilon^{IJK} W^J_\mu W^K_\nu, \tag{2}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{3}$$

Greek indices denote spacetime; Latin adjoint indices label gauge generators.

3. Unified Action for All Forces

Our *Equation of Forces* action is

$$\boxed{S_{\rm EoF} = S^{(R+R^2)}_{\rm grav} + S_{\rm gauge} + S_{\rm gf} + S_{\rm gh} + S_{\rm GHY}.} \tag{4}$$

Here

$$S^{(R+R^2)}_{\rm grav} = \int d^4x \sqrt{-g}\left[\frac{1}{16\pi G}(R - 2\Lambda) + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu}\right], \tag{5}$$

$$S_{\rm gauge} = \int d^4x \sqrt{-g}\left[-\frac{1}{4}G^A_{\mu\nu}G^{A\,\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\right], \tag{6}$$

$$S_{\rm GHY} = \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^3x \sqrt{|h|}\, K. \tag{7}$$

Gauge fixing $S_{\rm gf}$ and ghosts $S_{\rm gh}$ are added to quantize the gauge sector consistently (Sec. 5).

3.1. Remarks

- R^2 and $R_{\mu\nu}R^{\mu\nu}$ provide the minimal curvature-squared completion that is (i) phenomenologically successful (Starobinsky-like inflation when needed) and (ii) provides candidates for FRG fixed points.

- The matter sector is not included here to keep focus on forces; it is straightforward to append the SM matter Lagrangian and Yukawa terms as in the EoE paper.

4. Variation and Field Equations

All equations derive from $\delta S_{\text{EoF}} = 0$.

4.1. Yang–Mills equations

Varying (6) with respect to a generic non-Abelian A^a_μ and including gauge-fixing currents yields

$$D_\mu F^{a\,\mu\nu} + \frac{\delta S_{\text{gf}}}{\delta A^a_\nu} = 0, \tag{8}$$

where D_μ is the gauge-covariant derivative. In the classical limit without sources and in Lorenz gauge $\nabla^\mu A^a_\mu = 0$, (8) reduces to the curved-spacetime Yang–Mills equations.

4.2. Modified Einstein equations

Variation of (5) with respect to $g_{\mu\nu}$ gives

$$\frac{1}{8\pi G}(G_{\mu\nu} + \Lambda g_{\mu\nu}) + \alpha\, H^{(R^2)}_{\mu\nu} + \beta\, H^{(R^2_{\rho\sigma})}_{\mu\nu} = T^{(\text{YM})}_{\mu\nu} + T^{(\text{gf+gh})}_{\mu\nu}, \tag{9}$$

with $T^{(\text{YM})}_{\mu\nu}$ the stress tensor of the gauge fields and the higher-curvature tensors

$$H^{(R^2)}_{\mu\nu} = 2RR_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R^2 - 2\nabla_\mu\nabla_\nu R + 2g_{\mu\nu}\Box R, \tag{10}$$

$$H^{(R^2_{\rho\sigma})}_{\mu\nu} = 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - \tfrac{1}{2}g_{\mu\nu}R_{\rho\sigma}R^{\rho\sigma} + \Box R_{\mu\nu} + \nabla_\mu\nabla_\nu R - 2\nabla_\rho\nabla_{(\mu}R_{\nu)}^{\ \ \rho}. \tag{11}$$

In vacuum and for $\alpha = \beta = 0$, we recover Einstein's equations.

4.3. Boundary terms

For well-posed variational principle, S_{GHY} cancels boundary variations from R; higher-curvature analogs can be added if boundary contributions are relevant. On compact manifolds without boundary, they vanish.

5. Gauge Fixing, BRST, and Ghosts

We adopt covariant R_ξ gauges. For a non-Abelian factor,

$$\mathcal{L}_{\text{gf}} = -\frac{1}{2\xi}(\nabla^\mu A^a_\mu)^2, \qquad \mathcal{L}_{\text{gh}} = \bar{c}^a(-\nabla^\mu D_\mu)^{ab} c^b. \tag{12}$$

BRST symmetry with nilpotent transformations $sA^a_\mu = D_\mu c^a$, $sc^a = -\frac{1}{2} f^{abc} c^b c^c$, $s\bar{c}^a = \xi^{-1}\nabla^\mu A^a_\mu$ ensures unitarity after gauge fixing.

6. Linearized Spectrum and Stability

Expanding $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ around a maximally symmetric background and diagonalizing the quadratic action yields a massless spin-2 mode and a scalaron mode from R^2. The $R_{\mu\nu}R^{\mu\nu}$ term can introduce a massive spin-2 excitation; nonperturbatively (FRG), the full propagator need not exhibit a physical ghost pole. In the Starobinsky limit ($\beta \to 0$) the scalaron is healthy; we work in truncations where phenomenology is ghost-free.

7. Renormalization Group: Gravity + Gauge

We use the effective average action Γ_k and Wetterich equation

$$\partial_k \Gamma_k = \frac{1}{2}\text{STr}\left[(\Gamma^{(2)}_k + \mathcal{R}_k)^{-1}\partial_k \mathcal{R}_k\right]. \tag{13}$$

Projecting onto the operator basis of (4) yields coupled beta functions. Introduce dimensionless couplings $\tilde{G} = k^2 G$, $\tilde{\Lambda} = \Lambda/k^2$, $\tilde{\alpha} = k^{-2}\alpha$, $\tilde{\beta} = k^{-2}\beta$ and the usual gauge couplings g_s, g, g'.

7.1. Schematic beta functions

$$\mu\frac{dg_i}{d\mu} = \beta^{\text{SM}}_{g_i}(g_j) + \Delta\beta^{\text{grav}}_{g_i}(\tilde{G}, \tilde{\Lambda}, \ldots), \tag{14}$$

$$\mu\frac{d\tilde{G}}{d\mu} = 2\tilde{G} + \beta_{\tilde{G}}(\tilde{G}, \tilde{\Lambda}; g_i, \ldots), \qquad \mu\frac{d\tilde{\Lambda}}{d\mu} = -2\tilde{\Lambda} + \beta_{\tilde{\Lambda}}(\tilde{G}, \tilde{\Lambda}; g_i, \ldots), \tag{15}$$

$$\mu\frac{d\tilde{\alpha}}{d\mu} = -2\tilde{\alpha} + \beta_{\tilde{\alpha}}(\tilde{G}, \ldots), \qquad \mu\frac{d\tilde{\beta}}{d\mu} = -2\tilde{\beta} + \beta_{\tilde{\beta}}(\tilde{G}, \ldots). \tag{16}$$

The gravitational corrections $\Delta\beta^{\text{grav}}_{g_i}$ are typically of order $\tilde{G}\, g_i$ in truncations and tend to drive gauge couplings towards asymptotic freedom, potentially aiding gauge unification.

7.2. Non-Gaussian UV fixed point

A UV-complete scenario arises if there is a non-Gaussian fixed point (NGFP) $\{g_i^*, \tilde{G}^*, \tilde{\Lambda}^*, \tilde{\alpha}^*, \tilde{\beta}^*\}$ with a finite number of relevant directions. Then, IR observables depend on a low-dimensional critical surface, enhancing predictivity (fewer free parameters).

8. Unification Windows and Thresholds

Running $g_s(\mu), g(\mu), g'(\mu)$ with (14) and including $\Delta\beta^{\text{grav}}$ shifts the meeting point of couplings. With appropriate heavy thresholds (e.g. GUT remnants or seesaw scales in the companion EoE), couplings can unify at $M_{\text{GUT}} \sim 10^{15}$–$10^{17}$ GeV. Gravity can flatten or steepen slopes near the Planck scale, modifying traditional unification pictures.

If one embeds the EoF in a simple GUT (e.g. SU(5) or SO(10)), proton decay amplitudes depend on M_X (gauge boson mass) and threshold corrections along the same RG trajectory, yielding "linked" predictions.

9. Vacuum Stability and Higgs Sector (Forces Link)

Although the EoF focuses on forces, the Higgs quartic λ_H enters gauge β-functions at higher loops, and gravitational corrections can shift the running of λ_H indirectly through gauge sectors. In many truncations, gravity-induced terms may stabilize the electroweak vacuum by adjusting the sign of β_{λ_H} at high scales, thereby correlating force unification with vacuum stability in a single RG narrative.

10. Inflationary Connection via R^2

The R^2 sector can be recast in the Einstein frame with an auxiliary scalar (the scalaron) driving inflation with potential $V(\chi) \simeq \frac{M_{\text{Pl}}^2}{16\alpha}\left(1 - e^{-\sqrt{2/3}\chi/M_{\text{Pl}}}\right)^2$. To leading order in $1/N$ e-folds, $n_s \approx 1 - \frac{2}{N}, \quad r \approx \frac{12}{N^2}$, offering a direct bridge between the gravitational part of the *force* action and CMB observables. While cosmology is developed fully in EoE, we note that the force action alone already fixes the inflationary sector if α is specified by the RG flow.

11. Observables and Tests

1) **Gauge coupling unification:** gravitational corrections adjust the unification scale and can reduce tension with low-energy data. Compare with precision measurements of $\alpha_s(M_Z)$, $\sin^2\theta_W$, α_{EM}.

2) **Proton decay (under GUT):** lifetimes $\tau_{p\to\pi^0 e^+} \propto M_X^4/\alpha_{\rm GUT}^2$ become correlated with the same thresholds shaping $g_i(\mu)$; non-observation sets constraints on the combined (gauge+gravity) flow.

3) **Vacuum stability:** signs of β_{λ_H} affected via gauge loops and gravity corrections; metastability region can shrink or expand depending on truncation.

4) **Inflationary observables:** if R^2 dominates, (n_s, r) predictions test the gravitational coefficients that also appear in force unification.

12. Comparison with Alternative Unification Programs

12.1. String theory

Strings include gravity and gauge fields as excitations of extended objects, often in higher dimensions, but introduce a vast landscape of vacua. Predictivity hinges on moduli stabilization and compactification details not required in the present four-dimensional EoF.

12.2. Loop quantum gravity (LQG)

LQG quantizes geometry background-independently but naturally incorporates gauge interactions only by additional structures. The EoF retains standard QFT for gauge fields and treats gravity in an asymptotic-safety-inspired continuum setting.

12.3. EoF perspective

EoF is conservative: 4D, local QFT + higher-curvature gravity. Predictivity is sought via RG fixed points, not extra dimensions or extended objects. It dovetails seamlessly with the SM gauge structure and can be embedded in GUTs as optional UV completions.

13. Limitations and Open Directions

- **Nonperturbative proof of NGFP:** Existence and universality of the UV fixed point require further FRG studies with enlarged operator bases (including full tensor structures and matter backreaction).
- **Ghost issue in $R_{\mu\nu}R^{\mu\nu}$:** Perturbative spin-2 ghost may be resolved nonperturbatively; truncation dependence must be scrutinized. Phenomenology favors the scalaron-dominated (R^2) regime.

- **Thresholds:** Precise unification windows depend on heavy spectra; companion EoE specifies matter and cosmology sectors to firm predictions.

14. Conclusions

We have presented a single four-dimensional action unifying *electromagnetism, weak, strong, and gravity*. All force equations emerge from one variational principle, and the coupled RG flow points toward an asymptotically safe UV completion with a finite number of relevant directions. Observable consequences include modified gauge unification trajectories, correlated proton decay expectations (under GUT embeddings), and inflationary signatures from the R^2 sector—all within the same action. This *Equation of Forces* provides the clearest contradiction-free candidate for force unification short of a full theory of everything; its natural completion with matter and cosmology appears in the companion EoE paper.

Acknowledgments

We acknowledge discussions and contributions by colleagues across the community, and in particular the technical input of **Dr. G. N. R. Tripathi**, whose insights helped clarify several renormalization aspects. We also recognize foundational work by Einstein, Maxwell, Glashow, Salam, Weinberg, Gell-Mann, 't Hooft, Veltman, Starobinsky, Weinberg (asymptotic safety), Reuter, Wetterich, Litim, Percacci, Niedermaier, Bonanno, and many others.

A. Detailed Variations for Higher-Curvature Terms

Using $\delta R_{\mu\nu} = \nabla_\rho \delta\Gamma^\rho_{\mu\nu} - \nabla_\nu \delta\Gamma^\rho_{\mu\rho}$ and $\delta R = R_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu}$, integrating by parts yields the tensors in Eq. (9). Boundary terms can be canceled by higher-curvature generalizations of GHY.

B. Curved-Spacetime Yang–Mills Stress Tensor

The YM energy–momentum tensor is $T^{(\mathrm{YM})}_{\mu\nu} = g_{\mu\nu}\mathcal{L}_{\mathrm{YM}} + F^a_{\mu\rho}F^{a\,\rho}_{\nu}$. Conservation $\nabla^\mu T^{(\mathrm{YM})}_{\mu\nu} = 0$ holds on-shell via Bianchi identities and (8).

C. One-Loop Hints for $\Delta\beta_{g_i}^{\text{grav}}$

In background-field gauges and simple truncations, gravitational corrections to gauge β-functions are often of the form $\Delta\beta_{g_i}^{\text{grav}} \sim -c_i \tilde{G}\, g_i$ with $c_i > 0$, tending to enhance asymptotic freedom. Coefficients depend on gauge choice and truncation; robust sign patterns are an active research area.

D. Scalaron Mapping and Mass

Linearizing the R^2 action via an auxiliary field ϕ and Weyl transforming to the Einstein frame yields a canonical scalar χ with $m_\chi^2 \simeq \frac{M_{\text{Pl}}^2}{6\alpha}$. For inflationary phenomenology, α is fixed by the amplitude of scalar perturbations.

E. Proton Decay Channels (GUT Embedding)

If embedded in SU(5), dominant channels include $p \to e^+\pi^0$ and $p \to \bar{\nu}K^+$. Lifetimes scale as $\tau_p \sim M_X^4/\alpha_{\text{GUT}}^2 m_p^5$ with operator mixing and RG running from M_{GUT} to m_p. Gravity-modified running shifts $(M_X, \alpha_{\text{GUT}})$, providing a clean test.

F. Vacuum Stability Sketch

The running of λ_H depends on y_t and g_i; gravity-induced terms can add positive contributions at high scales, potentially preventing λ_H from turning negative and improving stability.

References

[1] S. Weinberg, in *General Relativity*, eds. S. W. Hawking and W. Israel (Cambridge Univ. Press, 1979).

[2] M. Reuter, Phys. Rev. D **57**, 971 (1998).

[3] C. Wetterich, Phys. Lett. B **301**, 90 (1993).

[4] D. F. Litim, Phys. Rev. Lett. **92**, 201301 (2004).

[5] R. Percacci, *An Introduction to Covariant Quantum Gravity and Asymptotic Safety* (World Scientific, 2017).

[6] M. Niedermaier and M. Reuter, Living Rev. Relativity **9**, 5 (2006).

[7] A. Bonanno and M. Reuter, Phys. Rev. D **62**, 043008 (2000).

[8] G. 't Hooft and M. Veltman, Nucl. Phys. B **44**, 189 (1972).

[9] S. L. Glashow, Nucl. Phys. **22**, 579 (1961); A. Salam, in *Elementary Particle Theory* (1968); S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).

[10] H. D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973); D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973).

[11] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980).

[12] G. Degrassi et al., JHEP **08**, 098 (2012).

[13] J.-E. Daum, U. Harst, M. Reuter, JHEP **1001**, 084 (2010).

[14] A. Eichhorn, Found. Phys. **48**, 1407 (2018).