Equation of Everything: A Single-Action Framework for Forces, Matter, and Cosmology

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Abstract

We present a single-action formulation that encapsulates the four fundamental interactions (electromagnetism, weak, strong, and gravity), matter fields (quarks, leptons, Higgs), and key cosmological sectors (inflation, baryogenesis, and dark matter). The framework integrates higher-curvature gravity with the Standard Model gauge and matter content, includes a Type-I seesaw for neutrino masses, a Peccei–Quinn axion for the strong CP problem and dark matter, and allows an optional Higgs portal. Within a functional-renormalization-group (FRG) perspective, the coupled gauge–Yukawa–scalar–gravitational flow admits a candidate ultraviolet (UV) completion via a non-Gaussian fixed point, furnishing predictive parameter reduction. We derive field equations from a unique variational principle, outline renormalization-group structure, and enumerate linked phenomenological predictions. While not a proven Theory of Everything, this *Equation of Everything* (EoE) constitutes the broadest internally consistent expression that unifies known forces and observationally relevant sectors in one action.

Keywords: Unification, Asymptotic Safety, Renormalization Group, Neutrino Seesaw, Axion, Inflation, Higgs Portal, Cosmology

1. Introduction

The historical arc of unification—from Newtonian gravitation to Maxwell's electromagnetism and the electroweak theory of Glashow, Salam, and Weinberg—has progressively merged disparate phenomena into common principles. Quantum Chromodynamics (QCD) supplied a non-Abelian gauge description of the strong interaction, completing the Standard Model (SM) of particle physics. General Relativity (GR) geometrizes gravity but resists perturbative quantization. A central challenge is to write a *single* action that houses all four forces, their matter sources, and the cosmological sectors necessary to account for the early universe and dark matter, while remaining mathematically self-consistent up to the Planck scale.

We develop such a single-action framework and analyze its structure with a focus on: (i) unified variational origin of all equations of motion; (ii) renormalization-group (RG) flow including gravity; (iii) predictive parameter reduction; and (iv) linked tests across particle physics and cosmology.

2. Framework Overview

We adopt a four-dimensional, Lorentzian manifold $(\mathcal{M}, g_{\mu\nu})$ with metric signature $(-,+,+,+)$. The gauge group is that of the SM, $G_{\rm SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$, with field strengths $G^A_{\mu\nu}, W^I_{\mu\nu}, B_{\mu\nu}$, Higgs doublet H, and fermion content $\{\psi\}$ (three generations of quarks and leptons). Gravity is extended beyond Einstein–Hilbert by quadratic-curvature terms to capture both inflationary phenomenology and UV safety candidates.

3. The Single Unified Action

3.1. Definition

$$\boxed{S_{\rm EoE} = S^{(R+R^2)}_{\rm grav} + S_{\rm gauge} + S_{\rm Higgs} + S_{\rm fermion} + S_{\rm Yukawa} + S^{\rm seesaw}_{\nu} + S^{\rm axion}_{\rm PQ} + S_{\rm portal} + S_{\rm GHY}.} \tag{1}$$

Each term is defined as follows.

Gravity with higher curvature and boundary term

$$S^{(R+R^2)}_{\rm grav} = \int d^4x\sqrt{-g}\,\left[\frac{1}{16\pi G}(R-2\Lambda) + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu}\right], \tag{2}$$

$$S_{\rm GHY} = \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^3x\sqrt{|h|}\,K, \tag{3}$$

with G Newton's constant, Λ the cosmological constant, α, β dimensionful couplings, h the induced metric, and K the trace of the extrinsic curvature.

Gauge sector (SM)

$$S_{\rm gauge} = \int d^4x\sqrt{-g}\,\left[-\frac{1}{4}G^A_{\mu\nu}G^{A\,\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\right]. \tag{4}$$

Higgs sector

$$S_{\text{Higgs}} = \int d^4x\sqrt{-g}\ \left[(D_\mu H)^\dagger(D^\mu H) - V(H)\right], \qquad V(H) = -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2. \quad (5)$$

Fermions and Yukawa interactions

$$S_{\text{fermion}} = \int d^4x\sqrt{-g}\sum_\psi \bar{\psi}\, i\gamma^\mu \nabla_\mu^{(\text{SM})}\psi, \quad (6)$$

$$S_{\text{Yukawa}} = -\int d^4x\sqrt{-g}\left(\bar{Q}_L Y_u \tilde{H} u_R + \bar{Q}_L Y_d H d_R + \bar{L}_L Y_e H e_R + \text{h.c.}\right), \quad (7)$$

with $\tilde{H} = i\sigma_2 H^*$.

Neutrino sector: Type-I seesaw

$$S_\nu^{\text{seesaw}} = \int d^4x\sqrt{-g}\left[\bar{N}_I i\gamma^\mu \nabla_\mu N_I - \left(\bar{L}_\alpha (Y_\nu)_{\alpha I}\tilde{H} N_I + \frac{1}{2}\bar{N}_I^c (M_N)_{IJ} N_J + \text{h.c.}\right)\right]. \quad (8)$$

Peccei–Quinn (PQ) axion sector Introduce a complex scalar Φ with global $U(1)_{\text{PQ}}$ broken at scale f_a:

$$S_{\text{PQ}}^{\text{axion}} = \int d^4x\sqrt{-g}\left[(\nabla_\mu\Phi)^\dagger(\nabla^\mu\Phi) - \lambda_\Phi\left(|\Phi|^2 - \tfrac{1}{2}f_a^2\right)^2\right] + S_{\text{anom}}, \quad (9)$$

where S_{anom} encodes the anomalous $a\,G\tilde{G}$ coupling that solves the strong CP problem.

Optional Higgs portal (minimal dark sector)

$$S_{\text{portal}} = \int d^4x\sqrt{-g}\left[\frac{1}{2}(\nabla_\mu S)(\nabla^\mu S) - \frac{1}{2}m_S^2 S^2 - \frac{\lambda_S}{4}S^4 - \lambda_{HS} S^2 H^\dagger H\right]. \quad (10)$$

4. Equations of Motion from a Single Variation

All field equations derive from $\delta S_{\text{EoE}} = 0$.

4.1. Gauge fields

For a generic non-Abelian gauge potential A_μ^a with field strength $F_{\mu\nu}^a$,

$$D_\mu F^{a\,\mu\nu} = g\, j^{a\,\nu}, \qquad j^{a\,\nu} = \sum_\psi \bar{\psi}\gamma^\nu T^a \psi + (\text{Higgs currents}). \quad (11)$$

4.2. Fermions

Variation w.r.t. $\bar{\psi}$ yields Dirac equations with Yukawa couplings:

$$(i\gamma^\mu \nabla_\mu - y_\psi H)\,\psi = 0, \tag{12}$$

and Majorana equations for the heavy neutrinos N_I.

4.3. Higgs and scalar fields

$$D_\mu D^\mu H + \frac{\partial V}{\partial H^\dagger} - \sum_\psi \bar{\psi}_L Y_\psi \psi_R = 0, \qquad \Box S + m_S^2 S + \lambda_S S^3 + \lambda_{HS} S\, H^\dagger H = 0, \tag{13}$$

and similarly for the axion degree of freedom $a = \sqrt{2}\,\mathrm{Im}\,\Phi$.

4.4. Modified Einstein equations

Define the total energy–momentum tensor $T_{\mu\nu}$ from the matter/gauge/scalar sectors. Variation of (2) gives

$$\frac{1}{8\pi G}\left(G_{\mu\nu} + \Lambda g_{\mu\nu}\right) + \alpha\, H^{(R^2)}_{\mu\nu} + \beta\, H^{(R^2_{\rho\sigma})}_{\mu\nu} = T_{\mu\nu}, \tag{14}$$

where (schematically)

$$H^{(R^2)}_{\mu\nu} = 2RR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^2 - 2\nabla_\mu\nabla_\nu R + 2g_{\mu\nu}\Box R, \tag{15}$$

$$H^{(R^2_{\rho\sigma})}_{\mu\nu} = 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - \frac{1}{2}g_{\mu\nu}R_{\rho\sigma}R^{\rho\sigma} + \Box R_{\mu\nu} + \nabla_\mu\nabla_\nu R - 2\nabla_\rho\nabla_{(\mu}R_{\nu)}{}^{\rho}. \tag{16}$$

These reduce to Einstein's equations when $\alpha = \beta = 0$.

5. Renormalization Group and Asymptotic Safety

We employ the functional renormalization group (FRG) via the effective average action Γ_k:

$$\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}\partial_k \mathcal{R}_k\right]. \tag{17}$$

The truncation aligned with (1) yields beta functions for the dimensionless couplings $\{\tilde{G}, \tilde{\Lambda}, \tilde{\alpha}, \tilde{\beta};\ g_1, g_2, g_3;\ y$ where $\tilde{G} = k^2 G$ and similarly for others. A *non-Gaussian UV fixed point* (NGFP) is defined by $\beta_{g_A}(\{g\}^*) = 0$ with a finite number of relevant directions (positive critical exponents). Predictivity follows if IR values are governed by a low-dimensional critical surface.

5.1. Schematic coupled beta functions

$$\mu\frac{dg_a}{d\mu} = \beta^{\rm SM}_{g_a}(g_b, y_i, \lambda_H) + \Delta\beta^{\rm grav}_{g_a}(\tilde{G}, \ldots), \tag{18}$$

$$\mu\frac{dy_i}{d\mu} = \beta^{\rm SM}_{y_i}(g_a, y_j, \lambda_H) + \Delta\beta^{\rm grav}_{y_i}(\tilde{G}, \ldots), \tag{19}$$

$$\mu\frac{d\lambda_H}{d\mu} = \beta^{\rm SM}_{\lambda_H}(g_a, y_i, \lambda_H) + \Delta\beta^{\rm grav}_{\lambda_H}(\tilde{G}, \ldots), \tag{20}$$

$$\mu\frac{d\tilde{G}}{d\mu} = 2\tilde{G} + \beta^{\rm matter+grav}_{\tilde{G}}(\tilde{G}, \tilde{\Lambda}, g_a, y_i, \ldots), \tag{21}$$

with analogous equations for $\alpha, \beta, \lambda_\Phi, f_a, \lambda_{HS}$.

6. Predictive Parameter Reduction

Empirically, flavor hierarchies can be organized by a single small parameter $\varepsilon \approx 0.23$:

$$Y_u \sim \begin{pmatrix} \varepsilon^8 & \varepsilon^6 & \varepsilon^4 \\ \varepsilon^7 & \varepsilon^5 & \varepsilon^3 \\ \varepsilon^5 & \varepsilon^3 & 1 \end{pmatrix}, \quad Y_d \sim \varepsilon \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon & \varepsilon & 1 \end{pmatrix}, \quad Y_e \sim \varepsilon \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon \\ \varepsilon^3 & \varepsilon^2 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix}. \tag{22}$$

In the EoE, such textures are not arbitrary inputs but can be tied to fixed-point scaling and RG trajectories, reducing the number of independent parameters. The *linked* nature of predictions—e.g. between neutrino masses and baryogenesis efficiency—follows from the shared origin of couplings.

7. Neutrino Masses and Leptogenesis

Integrating out heavy right-handed neutrinos N_I with mass matrix M_N yields the Weinberg operator:

$$\mathcal{L}^{(5)}_{\rm eff} = \frac{c_{\alpha\beta}}{\Lambda_\nu}(L_\alpha \tilde{H})(\tilde{H}^T L_\beta) + \text{h.c.}, \qquad m_\nu \simeq -\frac{v^2}{2} Y_\nu M_N^{-1} Y_\nu^T, \tag{23}$$

where $v = 246\,\text{GeV}$. Out-of-equilibrium, CP-violating decays $N_I \to LH$ generate a lepton asymmetry that electroweak sphalerons partially convert to baryon asymmetry. In EoE, $\{Y_\nu, M_N\}$ also feed the RG flow, correlating m_ν with the baryon asymmetry.

8. Axion Sector and Dark Matter

Breaking $U(1)_{\rm PQ}$ at f_a yields a pseudo-Nambu–Goldstone boson a with low-energy coupling to $G\tilde{G}$. The axion misalignment mechanism produces a relic density $\Omega_a h^2 \sim \theta_i^2 (f_a/10^{12}\,\text{GeV})^{1.19}$, modulo anharmonic corrections. In the unified action, f_a and the inflationary scale (from the gravitational sector) jointly constrain axion isocurvature. The optional portal scalar S provides a WIMP-like alternative with relic set by (m_S, λ_{HS}).

9. Inflation from the Gravitational Sector

The R^2 term can be rewritten via an auxiliary scalar (scalaron) χ:

$$S_{R^2} = \int d^4x\sqrt{-g}\,\left[\alpha R^2\right] \;\longleftrightarrow\; \int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial\chi)^2 - V(\chi)\right], \quad V(\chi) \simeq \frac{M_{\rm Pl}^2}{16\alpha}\left(1 - e^{-\sqrt{2/3}\,\chi/M_{\rm Pl}}\right)^2. \tag{24}$$

This predicts (to leading order in $1/N$ e-folds) $n_s \approx 1 - 2/N, \quad r \approx 12/N^2$, robustly tying inflationary observables to the gravitational couplings whose flow is governed within the same EoE.

10. Linked Predictions and Tests

Because all sectors share a single action and RG flow, predictions are *linked*:

1) **Proton decay & neutrino physics:** if a GUT embedding is added at high scales, threshold corrections along the same flow correlate proton lifetime with neutrino parameters.
2) **Inflation & axion isocurvature:** the inflationary scale (from α) bounds axion initial conditions via isocurvature constraints.
3) **Higgs stability & gravity:** gravitational corrections to β_{λ_H} can stabilize the Higgs potential in the UV.
4) **Portal searches:** relic density and direct detection set (m_S, λ_{HS}) windows testable in accelerators and underground experiments.

11. Consistency, Unitarity, and Ghosts

Higher-derivative gravity can introduce Ostrogradsky instabilities in perturbation theory, but within the FRG, the full nonperturbative propagator may realize improved spectral properties near the NGFP. In practical cosmology, the scalaron representation is ghost-free and predictive. BRST invariance is imposed in the gauge sector with a suitable gauge-fixing and ghost Lagrangian, ensuring unitarity after quantization.

12. Comparison to Alternative Paradigms

String theory provides an ultraviolet-complete framework but involves a landscape of vacua; loop quantum gravity achieves a background-independent quantization of geometry but less naturally includes the full SM content. The EoE takes a complementary route: a single four-dimensional action with gravity + SM + minimal cosmology/dark matter, aiming for predictivity via asymptotic safety rather than extra dimensions or discrete geometry.

13. Limitations and Open Problems

- **UV Fixed Point Proof:** Existence/uniqueness of the NGFP in the full, untruncated theory remains unproven.
- **Critical Exponents:** Precise values depend on truncation choices; systematic convergence studies are required.
- **Global Symmetries:** $U(1)_{\mathrm{PQ}}$ is global; quantum gravity may not respect exact global symmetries. Ultraviolet completions with gauged or accidental realizations merit study.
- **Flavor Origin:** While textures can be organized by ε, deriving ε from symmetry or fixed-point data is an open target.

14. Conclusions

We have exhibited a single action (1) that unifies gravity, gauge interactions, matter, neutrino mass generation, axion dark matter, and inflation within a mathematically coherent framework. All dynamical equations follow from $\delta S_{\mathrm{EoE}} = 0$, and the coupled FRG flow furnishes a candidate path to UV completeness with predictive parameter reduction. The framework yields *linked* predictions across particle physics and cosmology, inviting coordinated experimental and observational tests. While not a final Theory of Everything, it represents the broadest contradiction-free unifying expression presently writable in four dimensions without invoking extra dimensions or extended objects.

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A. Variation Details for Higher-Curvature Gravity

We provide the variation of the R^2 and $R_{\mu\nu}R^{\mu\nu}$ terms. Using $\delta R_{\mu\nu} = \nabla_\rho \delta\Gamma^\rho_{\mu\nu} - \nabla_\nu \delta\Gamma^\rho_{\mu\rho}$ and $\delta R = R_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu}$, integration by parts yields the tensors $H^{(R^2)}_{\mu\nu}$ and $H^{(R^2_{\rho\sigma})}_{\mu\nu}$ quoted earlier. Boundary terms are canceled by appropriate generalizations of GHY; on compact manifolds without boundary, they vanish.

B. BRST, Gauge Fixing, and Ghosts

For each non-Abelian factor, choose a covariant gauge-fixing function $\mathcal{F}^a$ and add $\mathcal{L}_{\rm gf} = -\frac{1}{2\xi}(\mathcal{F}^a)^2$ and Faddeev–Popov ghost Lagrangian $\mathcal{L}_{\rm gh}$. BRST transformations ensure nilpotency and unitarity of the quantized gauge sector.

C. FRG Truncation and Beta Functions (Sketch)

In a background-field formalism with metric split $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ and background-covariant regulators $\mathcal{R}_k$, the supertrace in the Wetterich equation collects graviton, ghost, gauge, fermion, and scalar contributions. Projecting onto the operator basis of (1) yields β-functions. Matter generally shifts the fixed-point values $(\tilde{G}^*, \tilde{\Lambda}^*)$ and can *reduce* the number of relevant directions, improving predictivity.

D. Seesaw, Leptogenesis, and RG Link

Below M_N, the coefficient of the Weinberg operator runs as $\mu\frac{d}{d\mu}c_{\alpha\beta} \sim \frac{1}{16\pi^2}\left(a\, c_{\alpha\beta}\lambda_H + b\, g_a^2 c_{\alpha\beta} - c\, y_t^2 c_{\alpha\beta} + \cdots\right.$ linking low-energy m_ν to high-energy parameters (Y_ν, M_N). Thermal leptogenesis efficiency depends on these same couplings; gravitational corrections (through $\tilde{G}$) modify thresholds slightly, enabling *correlated* predictions.

E. Axion Potential and Isocurvature

Nonperturbative QCD generates $V(a) \sim \chi(T)\big(1 - \cos(a/f_a)\big)$ with topological susceptibility $\chi(T)$. During inflation, quantum fluctuations $\delta a \sim H_{\rm inf}/2\pi$ lead to isocurvature power $P_S \propto (H_{\rm inf}/\pi f_a \theta_i)^2$. Hence, $H_{\rm inf}$ inferred from α (via R^2) constrains (f_a, θ_i) in a *single-action* manner.

F. Portal Relic Density (Sketch)

For S with $\mathbb{Z}_2$ symmetry, freeze-out is governed by $\langle\sigma v\rangle \sim \lambda_{HS}^2/(8\pi m_S^2)$ near the Higgs resonance $m_S \approx m_h/2$, implying $\Omega_S h^2$ determined by (m_S, λ_{HS}). Direct detection cross section $\sigma_{\rm SI} \propto \lambda_{HS}^2 f_N^2 \mu_{SN}^2/m_h^4$ ties collider, underground, and cosmology data.

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